Area Under The Curves

Question1

The area of the region bounded by the curves $y = x^3, y = x^2$ and the lines $x = 0$ and $x = 2$ is

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Options:

A.

$\frac{4}{3}$

B.

$\frac{3}{2}$

C.

$\frac{2}{3}$

D.

$\frac{5}{3}$

Answer: B

Solution:

Given, curves, $y = x^3, y = x^2$

for point of intersection

$$\begin{aligned} & x^3 = x^2 \\ \Rightarrow \quad & x^2(x-1) = 0 \Rightarrow x = 0, 1 \\ \therefore \quad & y = 0, 1 \end{aligned}$$

point of intersection are $(0, 0)$ and $(1, 1)$

∴ Required area

$$= \int_0^1 (x^2 - x^3) dx + \int_1^2 (x^3 - x^2) dx$$
$$= \left(\frac{x^3}{3} - \frac{x^4}{4}\right)_0^1 + \left(\frac{x^4}{4} - \frac{x^3}{3}\right)_1^2$$
$$= \left(\frac{1}{3} - \frac{1}{4}\right) + \left[\left(\frac{16}{4} - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)\right]$$
$$= \frac{1}{12} + \frac{15}{4} - \frac{7}{3}$$
$$= \frac{1 + 45 - 28}{12} = \frac{18}{12} = \frac{3}{2} \text{ sq. units}$$

Question2

The area of the region (in sq units) enclosed by the curves $y = 8x^3 - 1, y = 0, x = -1$ and $x = 1$ is

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Options:

A. $\frac{15}{4}$

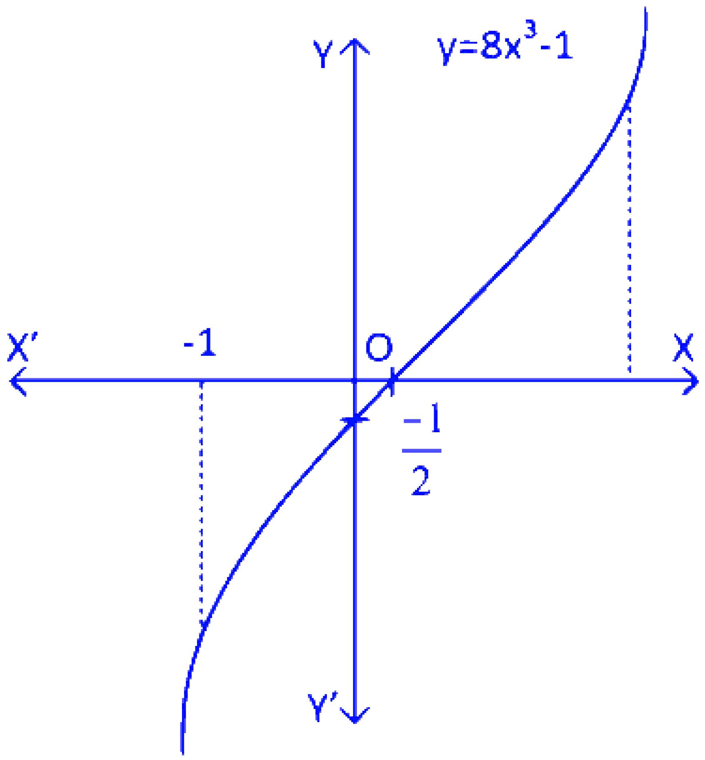
B. $\frac{15}{8}$

C. $\frac{19}{4}$

D. $\frac{19}{8}$

Answer: C

Solution:



$A=\int_{-1}^{\frac{1}{2}}\left(8x^3-1\right)dx+\int_{\frac{1}{2}}^{1}8x^3-1$

$A=\left(\frac{8x^4}{4}-x\right)_{-1}^{1/2}+\left(8\frac{x^4}{4}-x\right)_{1/2}^{1}$

$=\left|\left(2x^4-x\right)_{-1}^{1/2}\right|+\left(2x^4-x\right)_{1/2}^{1}$

$=\left|\left(2\cdot\frac{1}{16}-\frac{1}{2}\right)-(2+1)\right|+\left[(2-1)-\left(\frac{2}{16}-\frac{1}{2}\right)\right]$

$=\left|\frac{1}{8}-\frac{1}{2}-3\right|+1-\frac{1}{8}+\frac{1}{2}$

$=\left|\frac{1-4-24}{8}\right|+\frac{8-1+4}{8}$

$=\frac{27}{8}+\frac{11}{8}=\frac{38}{8}=\frac{19}{4}$

Question3

If the area of the region enclosed by the curve $ay=x^2$ and the line $x+y=2a$ is ka^2, then $k=$

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Options:

A. $\frac{2}{9}$

B. $\frac{9}{2}$

C. $\frac{3}{2}$

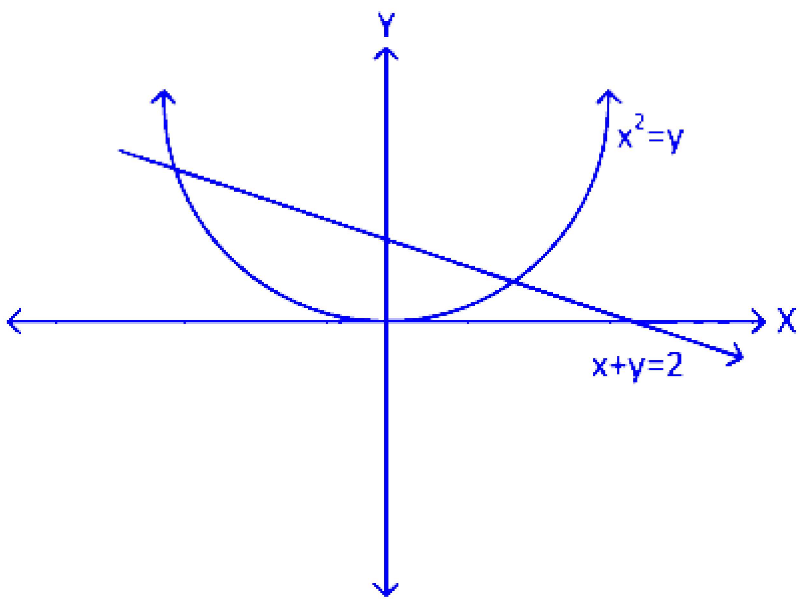
D. $\frac{2}{3}$

Answer: B

Solution:

Let the value of $a = 1$

then area between curves $x^2 = y$ and $x + y = 2$ is k



$$y = 2 - x$$

$$\Rightarrow \quad x^2 = 2 - x$$

$$x^2 + x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x = -2, x = 1$$

Area $= \int_{-2}^{1} (x^2 - 2 + x) dx$

$= \left[\frac{x^3}{3} - 2x + \frac{x^2}{2}\right]_{-2}^{1}$

$= \left(\frac{1}{3} - 2 + \frac{1}{2}\right) - \left(\frac{-8}{3} + 4 + 2\right)$

$= \frac{5}{6} - 2 + \frac{8}{3} - 6$

$= \frac{16+5}{6} - 8$

$= \frac{21}{6} - 8$

$= \frac{7}{2} - 8 = \frac{-9}{2}$

Area $= \frac{9}{2} = k$

Question4

The area of the region enclosed by the curves $y^2 = 4(x+1)$ and $y^2 = 5(x-4)$ is

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Options:

A. $\frac{280}{3}$

B. 150

C. 140

D. $\frac{200}{3}$

Answer: D

Solution:

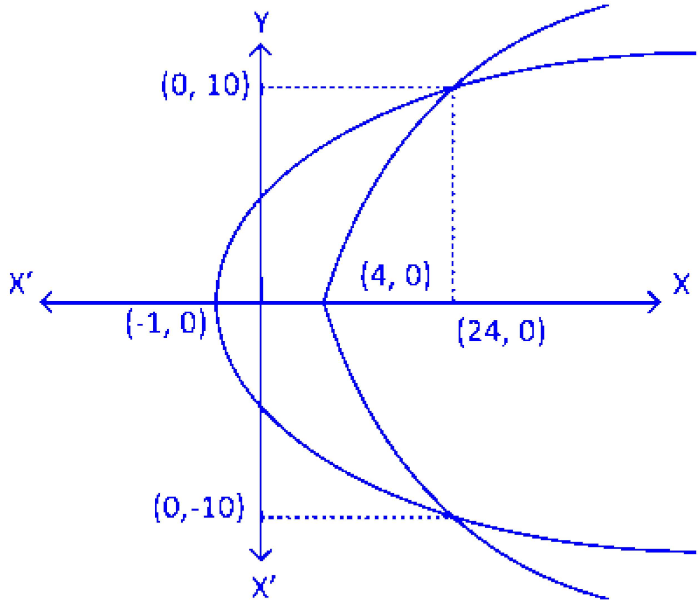
We have,

$y^2 = 4x + 4$ and $y^2 = 5x - 20$

$\Rightarrow 4x + 4 = 5x - 20$

$x = 24 \Rightarrow y^2 = 4(x+1)$

$y^2 = 4 \times 25 \Rightarrow y = \pm 10$



Shaded area

$= 2\left[\int_{-1}^{24} 2\sqrt{x+1} - \int_{4}^{24} \sqrt{5}\sqrt{x-4}\right]$

$= 2\left[2\left[\frac{(x+1)^{3/2}}{3/2}\right)_{-1}^{24} - \sqrt{5}\left(\frac{(x-4)^{3/2}}{3/2}\right)_{4}^{24}\right]$

$= 2\left[\frac{4}{3}(25)^{3/2} - \frac{2\sqrt{5}}{3}(\sqrt{20})^{3}\right]$

$= \frac{2}{3}[4 \times 125 - 2 \times \sqrt{5} \times 20 \times 2\sqrt{5}]$

$= \frac{2}{3}[500 - 400] = \frac{200}{3}$ sq. units

Question5

Area of the region enclosed between the curves $y^2 = 4(x+7)$ and $y^2 = 5(2-x)$ is

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Options:

A. $\frac{32\sqrt{2}}{3}$

B. $\frac{8}{3}$

C. $\frac{1}{6}$

D. $24\sqrt{5}$

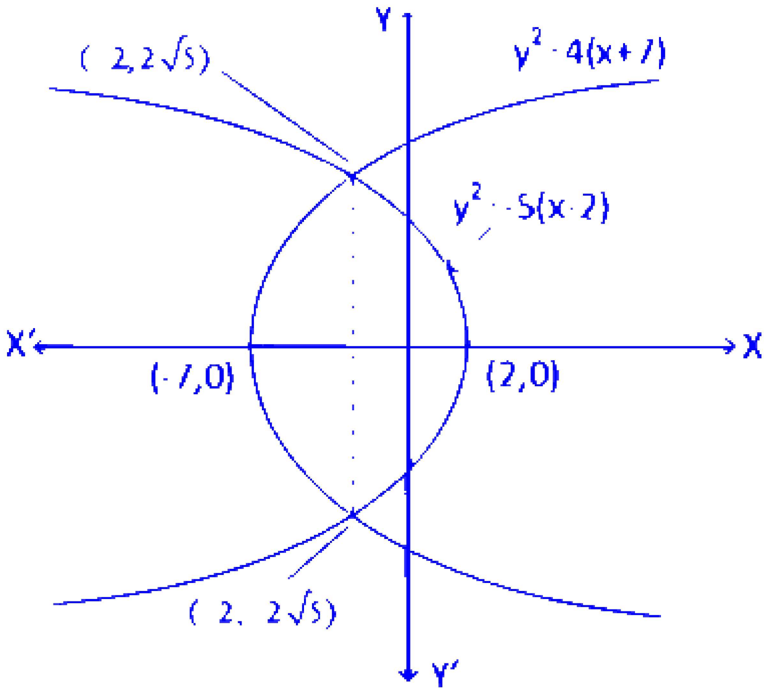
Answer: D

Solution:

We have,

$C_1 : y^2 = 4(x+7)$

$C_2 : y^2 = 5(2-x) = -5(x-2)$



$$\therefore \text{ Required area } = \left|\int (x_1 - x_2) dy\right|$$

$$= \left|\int_{-2\sqrt{5}}^{2\sqrt{5}} \left[\left(2 - \frac{y^2}{5}\right) - \left(\frac{y^2}{4} - 7\right)\right] dy\right|$$

$$= \left|\int_{-2\sqrt{5}}^{2\sqrt{5}} \left(-\frac{9y^2}{20} + 9\right) dy\right|$$

$$= \left|\left[-\frac{3y^3}{20} + 9y\right]_{-2\sqrt{5}}^{2\sqrt{5}}\right| = 24\sqrt{5}$$

Question6

The area (in sq units) bounded by the curve $y = 2x - x^2$ and the line $y = -x$ is

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Options:

A. $\frac{9}{2}$

B. $\frac{11}{2}$

C. $\frac{16}{3}$

D. $\frac{22}{5}$

Answer: A

Solution:

The required area is

$$= \int_0^3 \left\{\left(2x - x^2\right) - (-x)\right\} dx$$

$$= \int_0^3 \left(2x - x^2 + x\right) dx = \int_0^3 \left(3x - x^2\right) dx$$

$$= \left(\frac{3x^2}{2} - \frac{x^3}{3}\right)_0^3 = \frac{27}{2} - \frac{27}{3} - 0$$

$$= \frac{81 - 54}{6} = \frac{27}{6} = \frac{9}{2}$$
